sec 6

Binomial Random Variable

$$P(X) = \binom{n}{x} p^x q^{n-x} \quad (X = 0, 1, 2, \ldots n)$$

$\binom{n}{x} \rightarrow {}_nC_x$

X: is a binomial random variable that represent the no. of S's (→ success's) in n-trials.

n: No. of trials.

p: Probability of success in single trial.

q: 1-P (Probability of fail)

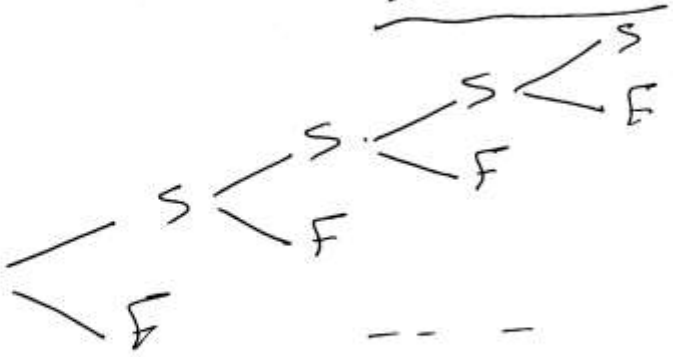
Ex: In an experiment 10% of people passes suppose that 4 persons are selected at random.

a) Find probability that none of the four persons passes the test.

b) _____ three of the four - - _____

c) Derive a formula for P(X), the probability distribution fn of the binomial random variable X.

Solution



Sample space

a) None passes $P(FFFF)$?

b) 3 Persons

SSSF, SFSS, FSSS, SSFS

(binomial) ← متعلقا بال

a) $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$p = 0.1$ $\quad q = 0.9$

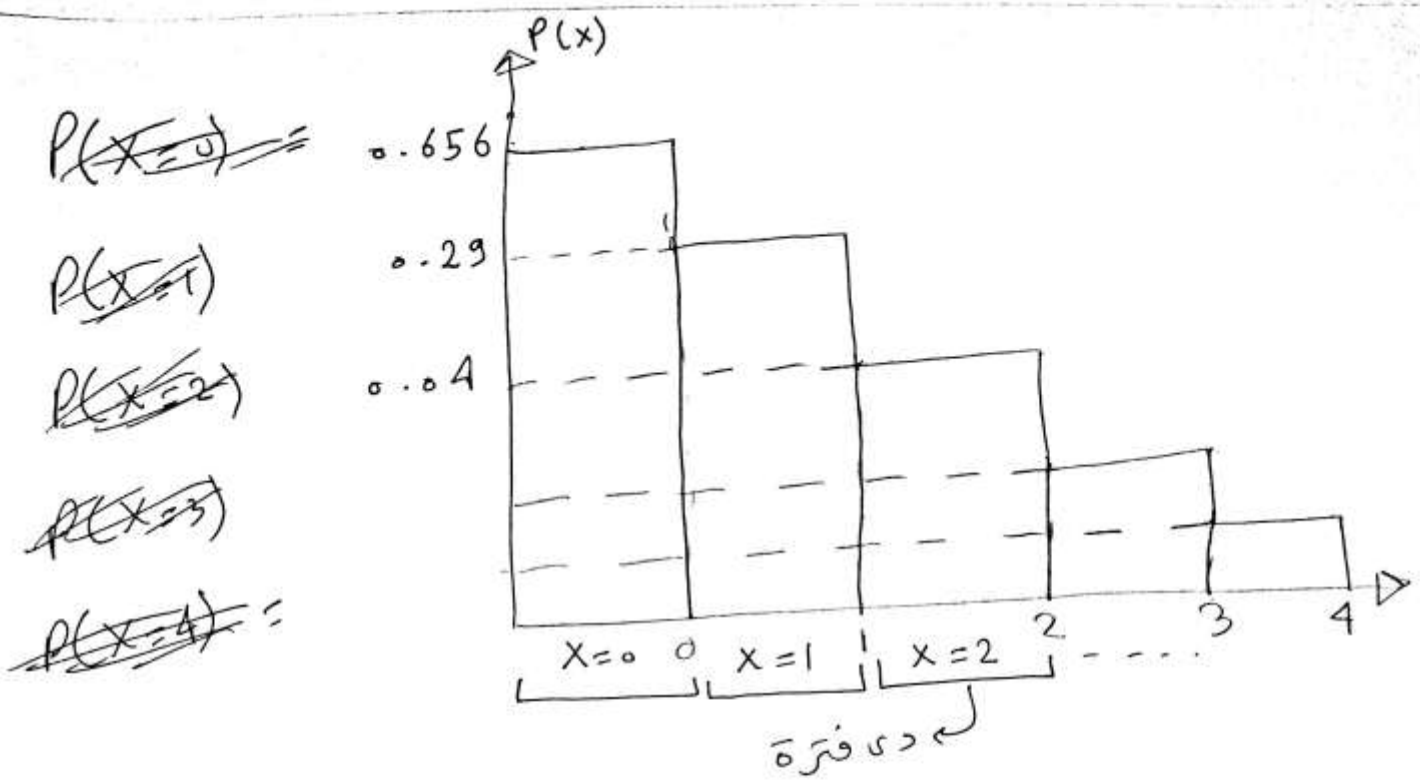
$x = 0$ $\quad n = 4$

$P(0) = {}^{4}C_{0}\, p^{0} q^{4}$

b) $P(3) = {}^{4}C_{3}\, p^{3} q^{1}$ ($p \to 0.1$, $q \to 0.9$)

c) Probability Distribution ←

X	0	1	2	3	4
P(X)	0.656	0.2916	0.0486	0.0036	0.0001



$\mu = np$

$Var = npq = a^2$

$S.D = a = \sqrt{npq}$

sheet 5

(3) Prove that for any random variable X

i) $E(aX+b) = aE(X) + b$

$$E(aX+b) = \int_{-\infty}^{\infty} (ax+b)\, p(x)\, dx$$

$$= a\underbrace{\int_{-\infty}^{\infty} x\, p(x)\, dx}_{E(x)} + b\underbrace{\int_{-\infty}^{\infty} p(x)\, dx}_{1}$$

$$E(aX+b) = a\,E(X) + b$$

7 Give a formula for $P(x)$ for a binomial random variable with $n = 7$

$$p = 0.2$$

Sol

$$P(x) = \binom{7}{x} (0.2)^x (0.8)^{7-x}$$

8 Consider the following

$$P(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x} \qquad (x = 0, 1, 2, \ldots)$$

a) n? p? b) Graph $P(x)$ c) Find μ, σ^2, σ

a) $p = 0.7$, $n = 5$, $q = 0.3$

b)

$P(X=0) =$

$P(X=1) =$

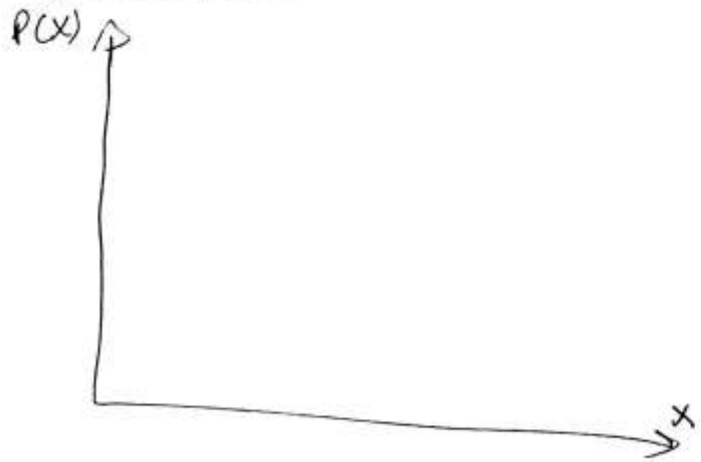
$P(X=2) =$

$P(X=3) =$

$P(X=4) =$

$P(X=5) =$

X	0	1	2	3	4	5
$P(x)$						



$$\mu = np = 5 * 0.7 =$$

$$S.D = \sqrt{npq}$$

$$Var = npq$$

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* The probability of exactly K success in n repeated trials.

$$b(K; n, P) = \binom{n}{K} p^K q^{n-K}$$

11 Fair Coin ⟶ 6 times

head ⟶ success

a) Probability that exactly 2 heads occur.

b) ⟶ at least 4 head

c) ⟵ no heads

d) ⟶ at least one head

Sol

a $P(X) = {}^{n}C_x \, p^x \, q^{n-x}$

$$P(2) = {}^{6}C_2 \, (0.5)^2 \, (0.5)^4$$

b) $k = 4, 5, 6$

$b(4; 6; 0{\cdot}5) + b(5; 6; 0{\cdot}5) + b(6; 6; 0{\cdot}5)$

d) $k = 1, 2, 3, \ldots 6$

$1 - b(0; 6; 0{\cdot}5)$

10 ⟶ report